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General Attitude Maneuvers of Spacecraft with Flexible Structures

Yiing-Yuh Lin* and Gern-Liang Lin†
National Cheng Kung University, Tainan, Taiwan, Republic of China

An efficient composite control strategy is proposed in this paper permitting simultaneous multi-axis reorientation of a spacecraft with flexible structures. The proposed control, separating large and small motions of the dynamic system, includes three parts that can be independently designed and tuned. For each maneuver, an open-loop referencing attitude control trajectory is derived by considering the spacecraft as rigid and executed by part 1. Flexural deformations of the structures are stabilized by part 2. Part 3, encompassing part 1 and part 2 control results and external disturbances, is an optimal guidance that tracks part 1 reference trajectory and achieves the specified final attitude in finite time. Three simulation cases are included in which a flexible model spacecraft is used. Collocated actuators and sensors are placed on the hub and on the flexible structures of the spacecraft.

Introduction

NEWLY designed or proposed satellites and space stations usually consist of a cluster of rigid units in the center and several lightweight mission structures, such as solar panels and antenna disks, attached to the sides of the center units through trusses.[1,2] To design an attitude control subsystem for these types of spacecraft is a challenging task and has been studied by various researchers.[3–10] In many approaches, the complex configurations of the spacecraft were usually simplified to a rigid hub with elastic appendages and control laws proposed were designed for single axis slewing maneuvers.[3–6] Some papers[7–10] formulated the system equations of the models for multi-axis attitude motions characterized by three successive Euler angle rotations; however, the examples included were basically single-axis slewing maneuvers. Quinn and Meirovitch[8] and Meirovitch and Kwak[9,10] proposed a control strategy demonstrated on planar motion examples that included an open-loop bang-bang-like rigid-body attitude control with feedback regulation of elastic vibrations and attitude perturbations. They basically sought a constant state feedback gain to regulate the perturbed motions by solving the algebraic Riccati equation. In effect, the coupled elastic motions and attitude perturbations were governed by a set of linear ordinary differential equations with time-varying coefficients associated to the open-loop trajectory. A general attitude reorientation can be carried out by following a proper sequence of three single-axis slewing maneuvers, but the total maneuver time may be long and unnecessary.[7] Attitude maneuver time can be shortened by commanding the spacecraft undergoing simultaneous multi-axis reorientation. Nevertheless, the difficulties involved are to design and to follow a suitable three-dimensional attitude trajectory and to quickly suppress the structure vibrations at the same time.

This paper proposes a three-part composite control strategy allowing simultaneous multi-axis attitude reorientation of a spacecraft with flexible structures. The objectives are to minimize control energy, to eliminate structure deflections, and to reach a specified final attitude over a given maneuver time. Most reported studies considered the attitude actuators on the hub only. The proposed control coordinates control actuators placed both on the hub and on the flexible structures to keep the flexible deflections and hence the associated attitude deviations small relative to the physical size and motion of the overall system during attitude maneuver. Under the control conditions, the formulated system equations were studied by perturbation method and order of magnitude analysis. The resulting nondimensional system equations were arranged into three sets of equations that are one way coupled from rigid-body motions to flexible-body vibrations and to attitude variations. Accordingly, each part of the three-part controller can be independently designed and tuned for rigid-body motions, structure vibrations, and perturbed attitude motions. To perform an attitude maneuver, an open-loop reference control trajectory is chosen first[11] to be executed by part 1. The trajectory can be constructed by ground control station or through parallel processor onboard[12] by regarding the spacecraft as rigid. Part 2 is a local output feedback control that utilizes collocated actuators and sensors on the structures to suppress the structural deflections. Part 3 is an optimal guidance that tracks the reference trajectory to meet the desired final orientation at a given final time. For practical application, reaction wheels, expansion jets, and momentum gyros can be used for controlling the attitude of the hub. Small reaction jets can be used for stabilizing the structures. Accelerometers and gyros are suitable for measuring translational and torsional motions of the structures. To balance the forces generated by the reaction jets on the structures, reaction jets on the hub will be activated to eliminate any translational acceleration due to structural control.

In many cases, the feedback control gain was obtained by solving the Riccati equation whether algebraic or differential. Two problems arise. If the gain is solved from the algebraic Riccati equation, that implies that the system will settle at infinite time. If a control system is considered for finite time maneuver, a large positive final penalty matrix is usually specified for computing the time-varying gain from the differential Riccati equation, which may cause the computing process to diverge. The trajectories solved by the aforesaid two approaches meet the final boundaries with errors. The errors are inherent due to the assumption in the linear quadratic regulator (LQR) formulation that the states are proportional to the costates. In this research, a numerical method, the method of particular solutions,[11] was applied to obtain the equivalent time-varying feedback gain, which takes the system exactly to the specified final boundary in finite time. Experiences indicate that this method is numerically stable as long as the integration method is stable (referring to the Appendix) and is free from the burden of artificially specifying large final penalty matrix.

Formulation of the System Equations

It is convenient to introduce the proposed composite control strategy through the procedure of modeling a spacecraft. Shown in Fig. 1 is the model used in this study, which consists of a rigid cylindrical hub in the middle and two solid bodies symmetrically connected to either side of the hub through uniformly distributed flexible beams. The system principal moments of inertia of the undeformed spacecraft are J_x, J_y, and J_z in terms of its body axis. The total mass of the spacecraft is M. To reduce the complexity of the system equations,

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*Associate Professor, Institute of Aeronautics and Astronautics.
†Graduate Research Assistant, Institute of Aeronautics and Astronautics.

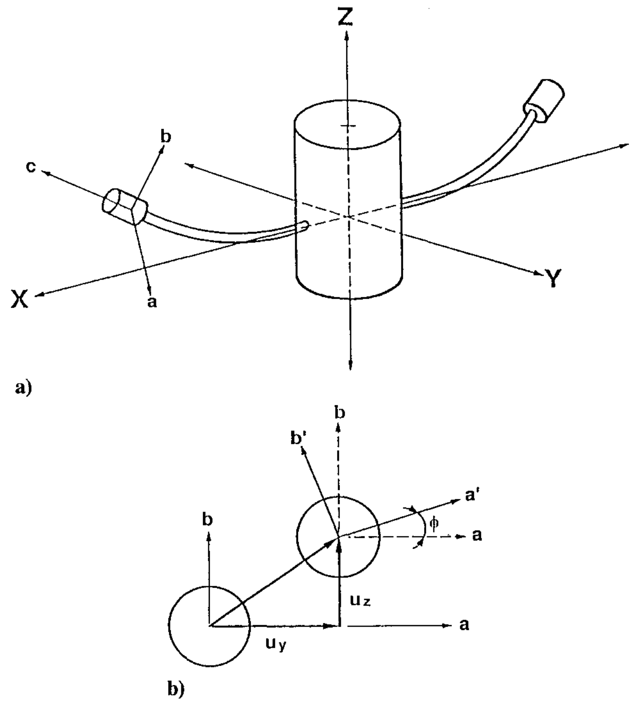

Fig. 1 Model spacecraft.

it is assumed that the two end bodies are identical with mass m_e, radius a, and the identical moments of inertia I_p. The two beams are considered to be uniform Euler–Bernoulli beams with the same length L, mass per unit length r, modulus of elasticity E, bending moment of inertia I, shear modulus G, torsional moment of inertia J, and torsional moment of inertia of unit length J_p.

Kinetic Equations of the System Rotational Motions

Consider the attitude motion of the spacecraft. We decouple the attitude motions from the translational motions by choosing the center of mass of the spacecraft as the origin of the body-fixed coordinates.[13] The velocity of a point p on the spacecraft is defined by

$$\boldsymbol{v}_p = \boldsymbol{\omega} \times (\boldsymbol{\xi}_p + \boldsymbol{u}) + \dot{\boldsymbol{u}} \tag{1}$$

where $\boldsymbol{\omega} = [\omega_x \ \omega_y \ \omega_z]$ is the angular velocity of the spacecraft in terms of its body frame; $\boldsymbol{\xi}_p$ is the position vector from the origin of the body frame to the undeformed point p; $\boldsymbol{u} = [u_{yi} \ u_{zi}]$ is the local deflection vector on the beam in y and z directions along body coordinates, and $i = 1, 2$, representing left or right beam. The kinetic energy of the system can be expressed as

$$\begin{aligned} T &= \frac{1}{2}\int_s \rho \boldsymbol{v}_p^{\mathrm{T}} \cdot \boldsymbol{v}_p \, \mathrm{d}S \\ &= \tfrac{1}{2}\boldsymbol{\omega}^{\mathrm{T}} \cdot K\boldsymbol{\omega} + \boldsymbol{\omega}\int_r^{r+L} \rho(\tilde{\xi}_p + \tilde{u})\dot{\boldsymbol{u}} \, \mathrm{d}x \\ &\quad + \frac{1}{2}\sum_{i=1}^{2}\int_r^{r+L} \left[\rho\dot{\boldsymbol{u}}^{\mathrm{T}} \cdot \dot{\boldsymbol{u}} + J_p(\omega_x + \dot{\phi}_i)^2\right] \mathrm{d}x \end{aligned} \tag{2}$$

where

$$K = \begin{bmatrix} J_x + \sum_{i=1}^{2}\int_r^{r+L} \rho\left(u_{yi}^2 + u_{zi}^2\right)\mathrm{d}x & -\sum_{i=1}^{2}\int_r^{r+L} \rho x u_{yi}\,\mathrm{d}x & -\sum_{i=1}^{2}\int_r^{r+L} \rho x u_{zi}\,\mathrm{d}x \\ -\sum_{i=1}^{2}\int_r^{r+L} \rho x u_{yi}\,\mathrm{d}x & J_y + \sum_{i=1}^{2}\int_r^{r+L} \rho u_{zi}^2\,\mathrm{d}x & -\sum_{i=1}^{2}\int_r^{r+L} \rho u_{zi}u_{yi}\,\mathrm{d}x \\ -\sum_{i=1}^{2}\int_r^{r+L} \rho x u_{zi}\,\mathrm{d}x & -\sum_{i=1}^{2}\int_r^{r+L} \rho u_{zi}u_{yi}\,\mathrm{d}x & J_z + \sum_{i=1}^{2}\int_r^{r+L} \rho u_{yi}^2\,\mathrm{d}x \end{bmatrix}$$

where $\tilde{\xi}_p$ and $\tilde{u}$ represent skew-symmetric matrices of the position vectors, and r is the radius of the hub. The strain energy of the beams is

$$\begin{aligned} U &= \frac{1}{2}\sum_{i=1}^{2}\int_r^{r+L} EI\left(u_{yi}''^2 + u_{zi}''^2\right)\mathrm{d}x + \frac{1}{2}\sum_{i=1}^{2}\int_r^{r+L} GJ\phi_i'^2\,\mathrm{d}x \\ &\quad + \frac{1}{2}\sum_{i=1}^{2}\int_r^{r+L}\int_x^{r+L} \rho\zeta\left(\omega_y^2 + \omega_z^2\right)\mathrm{d}\zeta\left(u_{yi}'^2 + u_{zi}'^2\right)\mathrm{d}x \end{aligned} \tag{3}$$

where the internal forces were ignored as the structure deflections are assumed small. The attitude dynamic model of the spacecraft was formulated through Lagrange's equations in terms of quasicoordinates as[14]

$$\begin{aligned} & J_x\dot{\omega}_x + \sum_{i=1}^{2}\Bigg(\int_r^{r+L} \rho\Big[\left(u_{yi}^2 + u_{zi}^2\right)\dot{\omega}_x - x u_{yi}\dot{\omega}_y \\ & - x u_{zi}\dot{\omega}_z - (u_{zi}\ddot{u}_{yi} - u_{yi}\ddot{u}_{zi})\Big] + J_p\ddot{\phi}_i\,\mathrm{d}x \\ & + \Big\{m_e\Big[\left(u_{yi}^2 + u_{zi}^2\right)\dot{\omega}_x - (L+a)u_{yi}\dot{\omega}_y - (L+a)u_{zi}\dot{\omega}_z \\ & - (u_{zi}\ddot{u}_{yi} - u_{yi}\ddot{u}_{zi})\Big] + I_e\ddot{\phi}_i\Big|_{L+a}\Big\}\Bigg) \\ & = -\omega_y\omega_z(J_z - J_y) + \sum_{i=1}^{2}\Bigg\{\int_r^{r+L} \rho\Big[\omega_x\omega_y x u_{zi} \\ & - \omega_x\omega_z x u_{yi} + \left(\omega_y^2 - \omega_z^2\right)u_{yi}u_{zi} - \omega_y\omega_x\left(u_{yi}^2 - u_{zi}^2\right) \\ & - 2\omega_x(u_{yi}\dot{u}_{yi} + u_{zi}\dot{u}_{zi})\Big]\mathrm{d}x + m_e\Big[\omega_x\omega_y(L+a)u_{zi} \\ & - \omega_x\omega_z(L+a)u_{yi} + \left(\omega_y^2 - \omega_z^2\right)u_{yi}u_{zi} \\ & - \omega_y\omega_x\left(u_{yi}^2 - u_{zi}^2\right) - 2\omega_x(u_{yi}\dot{u}_{yi} + u_{zi}\dot{u}_{zi})\Big]\Big|_{L+a}\Bigg\} + T_x \end{aligned} \tag{4}$$

$$\begin{aligned} & J_y\dot{\omega}_y + \sum_{i=1}^{2}\Bigg\{\int_r^{r+L} \rho\left(u_{zi}^2\dot{\omega}_y - x u_{yi}\dot{\omega}_x - u_{yi}u_{zi}\dot{\omega}_z - x\ddot{u}_{zi}\right)\mathrm{d}x \\ & + m_e\Big[u_{zi}^2\dot{\omega}_y - (L+a)(u_{yi}\dot{\omega}_x + \ddot{u}_{zi}) - u_{yi}u_{zi}\dot{\omega}_z\Big]\Big|_{L+a}\Bigg\} \\ & = -\omega_z\omega_x(J_x - J_z) + \sum_{i=1}^{2}\Bigg(\int_r^{r+L} \rho\Big[\omega_z\omega_y x u_{yi} \\ & - \omega_x\omega_y u_{zi}u_{yi} - \left(\omega_x^2 - \omega_z^2\right)x u_{zi} - \omega_z\omega_x u_{zi}^2 + 2\omega_x x\dot{u}_{yi} \\ & - 2\omega_y u_{zi}\dot{u}_{zi} + 2\omega_z\dot{u}_{yi}u_{zi}\Big] + J_p\omega_z\dot{\phi}_i\,\mathrm{d}x \\ & + \Big\{m_e\Big[\omega_z\omega_y(L+a)u_{yi} - \omega_x\omega_y u_{yi}u_{zi} \\ & - \left(\omega_x^2 - \omega_z^2\right)(L+a)u_{zi} - \omega_z\omega_x u_{zi}^2 + 2\omega_x(L+a)\dot{u}_{yi} \\ & - 2\omega_y u_{zi}\dot{u}_{zi} + 2\omega_z\dot{u}_{yi}u_{zi}\Big] + I_e\omega_z\dot{\phi}_i\Big|_{L+a}\Big\}\Bigg) + T_y \end{aligned} \tag{5}$$

$$
\begin{aligned}
&J_z\dot\omega_z + \sum_{i=1}^{2}\Bigg\{\int_r^{r+L}\rho\left(u_{yi}^2\dot\omega_z - xu_{zi}\dot\omega_x - u_{yi}u_{zi}\dot\omega_y + x\ddot u_{yi}\right)\mathrm{d}x\\
&+m_e\left[u_{yi}^2\dot\omega_z - (L+a)(u_{zi}\dot\omega_x + \ddot u_{yi}) - u_{zi}u_{yi}\dot\omega_y\right]\Big|_{L+a}\Bigg\}\\
&= -\omega_x\omega_y(I_y - I_z) + \sum_{i=1}^{2}\Bigg(\int_r^{r+L}\rho\big[-\omega_z\omega_y x u_{zi}\\
&+\omega_x\omega_z u_{zi}u_{yi} + \left(\omega_x^2 - \omega_y^2\right)xu_{yi} - \omega_y\omega_x u_{yi}^2 + 2\omega_x x\dot u_{zi}\\
&+2\omega_y u_{yi}\dot u_{zi} - 2\omega_z\dot u_{yi}u_{yi}\big] + J_p\omega_y\dot\phi_i\,\mathrm{d}x\\
&+\Big\{m_e\big[-\omega_z\omega_y(L+a)u_{zi} + \omega_x\omega_z u_{zi}u_{yi}\\
&+\left(\omega_x^2 - \omega_y^2\right)(L+a)u_{yi} - \omega_y\omega_x u_{yi}^2\\
&+2\omega_x(L+a)\dot u_{zi} + 2\omega_y u_{yi}\dot u_{zi} - 2\omega_z\dot u_{yi}u_{yi}\big]\\
&+I_e\omega_y\dot\phi_i\big|_{L+a}\Big\}\Bigg) + T_z
\end{aligned}
\tag{6}
$$

The attitude torques in Eqs. (4–6) exerted on the hub can be expressed as

$$T_x = T_{xe} + T_{xd} - \dot h_x + h_y\omega_z - h_z\omega_y + \sum_{i=1}^{2}\int_r^{r+L} T_{ti}\,\mathrm{d}x \tag{7}$$

$$T_y = T_{ye} + T_{yd} - \dot h_y + h_z\omega_x - h_x\omega_z + \sum_{i=1}^{2}\int_r^{r+L} xF_{zi}\,\mathrm{d}x \tag{8}$$

$$T_z = T_{ze} + T_{zd} - \dot h_z + h_x\omega_y - h_y\omega_x + \sum_{i=1}^{2}\int_r^{r+L} xF_{yi}\,\mathrm{d}x \tag{9}$$

where T_{je}, $j = x, y, z$ is the external control torque from attitude control jet or magnetic coil, T_{jd} is the environmental torque, and h_j is the angular momentum of the reaction wheel on the hub along each body axis. The term T_{ti} is the distributed control torque along each beam, and F_{yi} and F_{zi} are the distributed control forces on the beams along y and z body axes. The environmental torques and the torques produced by controlling the flexible beams are at least one order of magnitude smaller than the torques from the jet or the reaction wheels[16] and are considered as disturbance torques. To eliminate excessive translational motions of the hub caused by the resultant control forces vector $\boldsymbol{F}_b$ on the beams, a counteracting force vector $\boldsymbol{F}_h$ on the hub is needed to ensure $\boldsymbol{F}_h + \boldsymbol{F}_b \simeq 0$. The vector $\boldsymbol{F}_h$ can be provided by the expansion jets on the hub. The Euler parameters are used to describe the kinematics of the hub as $\beta_0 = \cos\theta/2$, $\beta_1 = l_1\sin\theta/2$, $\beta_2 = l_2\sin\theta/2$, $\beta_3 = l_3\sin\theta/2$, where $\boldsymbol{l} = [l_1\ l_2\ l_3]^{\mathrm{T}}$ is the principal line and θ is the principal angle of rotation.[15] The kinematical differential equation of the Euler parameters is expressed as

$$\dot\beta(t) = \{G[\omega(t)]\}\beta(t) \tag{10}$$

where

$$\{G[\omega(t)]\} = \frac{1}{2}\begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3\\ \omega_1 & 0 & \omega_3 & -\omega_2\\ \omega_2 & -\omega_3 & 0 & \omega_1\\ \omega_3 & \omega_2 & -\omega_1 & 0\end{bmatrix}$$

$$\sum_{k=0}^{3}\beta_k^2 = 1$$

The equations of the structural deflections for the left or the right beam in y and z directions are

$$
\begin{aligned}
&\rho\ddot u_{yi} - \rho u_{zi}\dot\omega_x + \rho x\dot\omega_z = \rho\left(\omega_x^2 + \omega_z^2\right)u_{yi}\\
&\quad - \rho x\omega_y\omega_x - \rho u_{zi}\omega_y\omega_x + 2\rho\omega_x\dot u_{zi} - \mathcal{L}u_{yi} + F_{yi}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&\rho\ddot u_{zi} + \rho u_{yi}\dot\omega_x - \rho x\dot\omega_y = \rho\left(\omega_x^2 + \omega_y^2\right)u_{zi}\\
&\quad - \rho x\omega_z\omega_x - \rho u_{yi}\omega_y\omega_x - 2\rho\omega_x\dot u_{yi} - \mathcal{L}u_{zi} + F_{zi}
\end{aligned}
\tag{12}
$$

where the $\mathcal{L}$ operator in Eqs. (11) and (12) is[14]

$$\mathcal{L} = \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2}{\partial x^2}\right) - \frac{\partial}{\partial x}\left[\int_x^{r+L}\rho\left(\omega_y^2 + \omega_z^2\right)\zeta\,\mathrm{d}\zeta\frac{\partial}{\partial x}\right]$$

and EI is assumed the same along y and z axes. The equation of torsional motions for the right or left side of beam is

$$J_p(\ddot\phi_i + \dot\omega_x) - GJ\frac{\partial^2\phi_i}{\partial x^2} = T_{ti} \tag{13}$$

where ϕ_i, $i = 1, 2$, is the torsional deflections on either side of beam. Equations (4–6) and (10–13) derived earlier describe the coupled rigid and flexible-body motions of the system.

Order of Magnitude Analysis

Equations (4–6) and (10–13) were made nondimensional first, and a small bookkeeping term ϵ, where $0 < \epsilon \ll 1$, was introduced to the nondimensional states as $u_{ji}/L = \epsilon u_{ji}^1 + \cdots$, $\omega_j/\Omega = \omega_j^0 + \epsilon\omega_j^1 + \cdots$, $\phi = \epsilon\phi_i^1 + \cdots$, $x/L = \bar x$, $a/L = \bar a$, $i = 1, 2$, $j = x, y, z$, where L is the length of the beam, and Ω is the nominal attitude rate of the spacecraft during reorientation maneuver. Equations (4–6) and (10–13) were separated into order of ϵ^0 and ϵ^1 by the substitution of the nondimensional states and keeping only to the order of ϵ^1. The terms that involved time derivative were also made nondimensional by defining $\tau = \Omega t$, where $\mathrm{d}/\mathrm{d}t = \Omega\mathrm{d}/\mathrm{d}\tau$ and $\mathrm{d}^2/\mathrm{d}t^2 = \Omega^2\mathrm{d}^2/\mathrm{d}\tau^2$. To simplify the notations, we use the same $(\dot{*})$ and $(\ddot{*})$ as in the previous equations to represent the nondimensional differentiations in the following equations. The equations of order ϵ^0 representing rigid-body motions are

$$\dot\omega_x^0 = -(J_z - J_y)\omega_y^0\omega_z^0/J_x + T_x^0/J_x\Omega^2 \tag{14}$$

$$\dot\omega_y^0 = -(J_x - J_z)\omega_z^0\omega_x^0/J_y + T_y^0/J_y\Omega^2 \tag{15}$$

$$\dot\omega_z^0 = -(J_y - J_x)\omega_x^0\omega_y^0/J_z + T_z^0/J_z\Omega^2 \tag{16}$$

The equations of order ϵ^1 for the deviations of attitude motions are

$$
\begin{aligned}
&\dot\omega_x^1 + \sum_{i=1}^{2}\Bigg\{\int_0^1\left(\frac{\rho L^3}{J_x}\right)\bar x\left(-u_{yi}^1\dot\omega_y^0 - u_{zi}^1\dot\omega_z^0\right) + \frac{J_pL}{J_x}\ddot\phi_i^1\,\mathrm{d}\bar x\\
&+\left[\left(\frac{m_eL^2}{J_x}\right)(1+\bar a)\left(-u_{yi}^1\dot\omega_y^0 - u_{zi}^1\dot\omega_z^0\right) + \left(\frac{J_e}{J_x}\right)\ddot\phi_i^1\bigg|_{1+\bar a}\right]\Bigg\}\\
&= -\left(\frac{J_z - J_y}{J_x}\right)\left(\omega_y^0\omega_z^1 + \omega_z^0\omega_y^1\right) + \sum_{i=1}^{2}\Bigg[\int_0^1\left(\frac{\rho L^3}{J_x}\right)\\
&\times\bar x\left(\omega_x^0\omega_y^0u_{zi}^1 - \omega_x^0\omega_z^0u_{yi}^1\right)\mathrm{d}\bar x + \left(\frac{m_eL^2}{J_x}\right)(1+\bar a)\\
&\times\left(\omega_x^0\omega_y^0u_{zi}^1 - \omega_x^0\omega_z^0u_{yi}^1\right)\Big|_{1+\bar a}\Bigg] + T_x^1\left(\frac{1}{J_x\Omega^2}\right)
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&\dot\omega_y^1 + \sum_{i=1}^{2}\Bigg[\int_0^1\left(\frac{\rho L^3}{J_y}\right)\bar x\left(-u_{yi}^1\dot\omega_x^0 - \ddot u_{zi}^1\right)\mathrm{d}\bar x\\
&+\left(\frac{m_eL^2}{J_y}\right)(1+\bar a)\left(-u_{yi}^1\dot\omega_x^0 - \ddot u_{zi}^1\right)\Big|_{1+\bar a}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&= -\left(\frac{J_x - J_z}{J_y}\right)\left(\omega_z^0\omega_x^1 + \omega_x^0\omega_z^1\right) \\
&+ \sum_{i=1}^{2}\Bigg\{\int_0^1 \left(\frac{\rho L^3}{J_y}\right)\bar{x}\big[\omega_z^0\omega_y^0 u_{yi}^1 - \left(\omega_x^{0^2} - \omega_z^{0^2}\right)u_{zi}^1 \\
&+ 2\omega_x^0\dot{u}_{yi}^1\big] + \left(\frac{J_p}{J_y}\right)\omega_z^0\dot{\phi}_i^1\,\mathrm{d}\bar{x} + \left(\left(\frac{m_e L^2}{J_z}\right)(1+\bar{a})\right. \\
&\times\big[\omega_z^0\omega_y^0 u_{yi}^1 - \left(\omega_x^{0^2} - \omega_z^{0^2}\right)u_{zi}^1 + 2\omega_x^0\dot{u}_{yi}^1\big] \\
&+ \left(\frac{J_e}{J_y}\right)\omega_z^0\dot{\phi}_i^1\Big|_{1+\bar{a}}\Bigg\}\Bigg) + T_y^1\left(\frac{1}{J_y\Omega^2}\right)
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\dot{\omega}_z^1 + \sum_{i=1}^{2}\Bigg[\int_0^1 \left(\frac{\rho L^3}{J_z}\right)\bar{x}\left(-u_{zi}^1\dot{\omega}_x^0 + \ddot{u}_{yi}^1\right)\mathrm{d}\bar{x} \\
&+ \left(\frac{m_e L^2}{J_z}\right)(1+\bar{a})\left(-u_{zi}^1\dot{\omega}_x^0 - \ddot{u}_{yi}^1\right)\Big|_{1+\bar{a}}\Bigg] \\
&= -\left(\frac{J_y - J_x}{J_z}\right)\left(\omega_y^0\omega_x^1 + \omega_x^0\omega_y^1\right) + \sum_{i=1}^{2}\left(\int_0^1\left(\frac{\rho L^3}{J_z}\right)\right. \\
&\times\bar{x}\big[-\omega_y^0\omega_z^0 u_{zi}^1 + \left(\omega_x^{0^2} - \omega_y^{0^2}\right)u_{yi}^1 - 2\omega_x^0\dot{u}_{zi}^1\big] \\
&- \left(\frac{J_p}{J_z}\right)\omega_y^0\dot{\phi}_i^1\,\mathrm{d}\bar{x} + \Bigg\{\left(\frac{m_e L^2}{J_z}\right)(1+\bar{a}) \\
&\times\big[-\omega_y^0\omega_z^0 u_{zi}^1 + \left(\omega_x^{0^2} - \omega_y^{0^2}\right)u_{yi}^1 - 2\omega_x^0\dot{u}_{zi}^1\big] \\
&- \left(\frac{J_e}{J_z}\right)\omega_y^0\dot{\phi}_i^1\Big|_{1+\bar{a}}\Bigg\}\Bigg) + T_z^1\left(\frac{1}{J_z\Omega^2}\right)
\end{aligned}
\tag{19}
$$

where the attitude torques are defined as $T_j = T_j^0 + \epsilon T_j^1 + \cdots$. Attitude deviations may be caused by structural deformations or other disturbances. The kinematic equations are also separated into

$$\dot{\beta}^0(t) = \{G[\omega^0(t)]\}\beta^0(t) \tag{20}$$

$$\dot{\beta}^1(t) = \{G[\omega^0(t)]\}\beta^1(t) + \{G[\omega^1(t)]\}\beta^0(t) \tag{21}$$

where $\sum_{k=0}^{3}(\beta_k^0 + \epsilon\beta_k^1)^2 = 1$. Equation (20) represents the nominal attitude motions and Eq. (21) describes the perturbed attitude motions. Beam deflections are considered small in the order of ϵ^1, and Eqs. (11) and (12) are made nondimensional as

$$
\begin{aligned}
\ddot{u}_{yi}^1 &= -\bar{x}\left(\dot{\omega}_z^0 + \omega_y^0\omega_x^0\right) + 2\dot{u}_{zi}^1\omega_x^0 + u_{zi}^1\left(\dot{\omega}_x^0 - \omega_y^0\omega_z^0\right) \\
&+ u_{yi}^1\left(\omega_x^{0^2} + \omega_z^{0^2}\right) - \frac{1}{\rho L\Omega^2}\left(\mathcal{L}u_{yi}^1 - F_{yi}\right)
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\ddot{u}_{zi}^1 &= \bar{x}\left(\dot{\omega}_y^0 - \omega_z^0\omega_x^0\right) - 2\dot{u}_{yi}^1\omega_x^0 - u_{yi}^1\left(\dot{\omega}_x^0 + \omega_z^0\omega_y^0\right) \\
&+ u_{zi}^1\left(\omega_x^{0^2} + \omega_y^{0^2}\right) - \frac{1}{\rho L\Omega^2}\left(\mathcal{L}u_{zi}^1 - F_{zi}\right)
\end{aligned}
\tag{23}
$$

The equations of torsional motions for both sides of the beam are

$$\ddot{\phi}_i^1 = -\dot{\omega}_x^0 - \left(\frac{1}{J_p\Omega^2}\right)\left(GJ\frac{\partial^2\phi_i^1}{\partial x^2} - T_{ti}\right) \tag{24}$$

Angular rate components in Eqs. (22–24) were kept to the order of ω_j^0 with the order of ω_j^1 neglected as the latter are one order smaller than the former. Note that Eqs. (14–16) of rigid-body motions are one way coupled to Eqs. (17–19) and Eqs. (22–24), and Eqs. (22–24) of flexural vibrations are also one way coupled to Eqs. (17–19) of attitude deviations.

Attitude Control and Structure Stabilization

Because the three sets of equations (14–24) are formulated as one way coupling from rigid-body motion to structure vibrations and to attitude deviations, the control for each set can be independently designed and tuned. An attitude control strategy with three parts is proposed: part 1 maneuvers the system by executing a reference control trajectory, part 2 suppresses flexible beam deformations caused by attitude motions, and part 3, taking into account the former two control effects, eliminates attitude deviations and guides the system to the desired final boundary by following the reference trajectory.

Open-Loop Nominal Control Trajectory

The simultaneous three-axis attitude trajectory for a spacecraft to move from one end of boundary to the other is not unique, and the problem of finding such a trajectory has been studied by many researches.[11,12,15] A suitable attitude trajectory can be determined by ground control station or using a parallel processor onboard the spacecraft. In this paper, we consider the objective of optimally re-

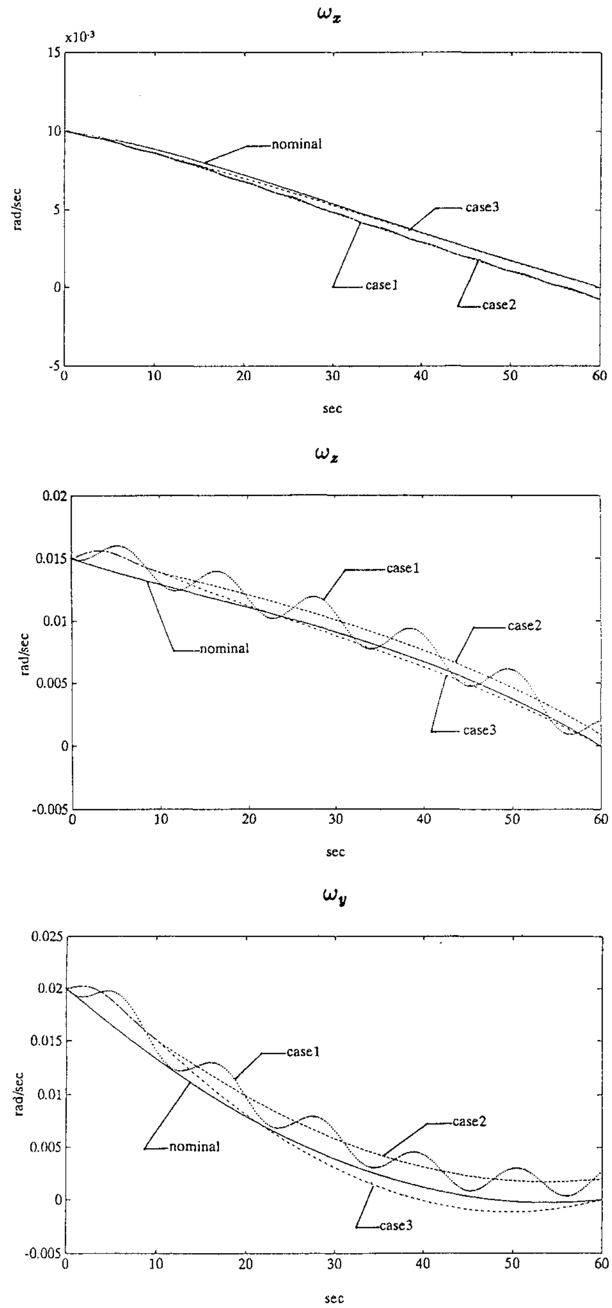

Fig. 2 Angular velocity components of nominal, case 1, case 2, and case 3.

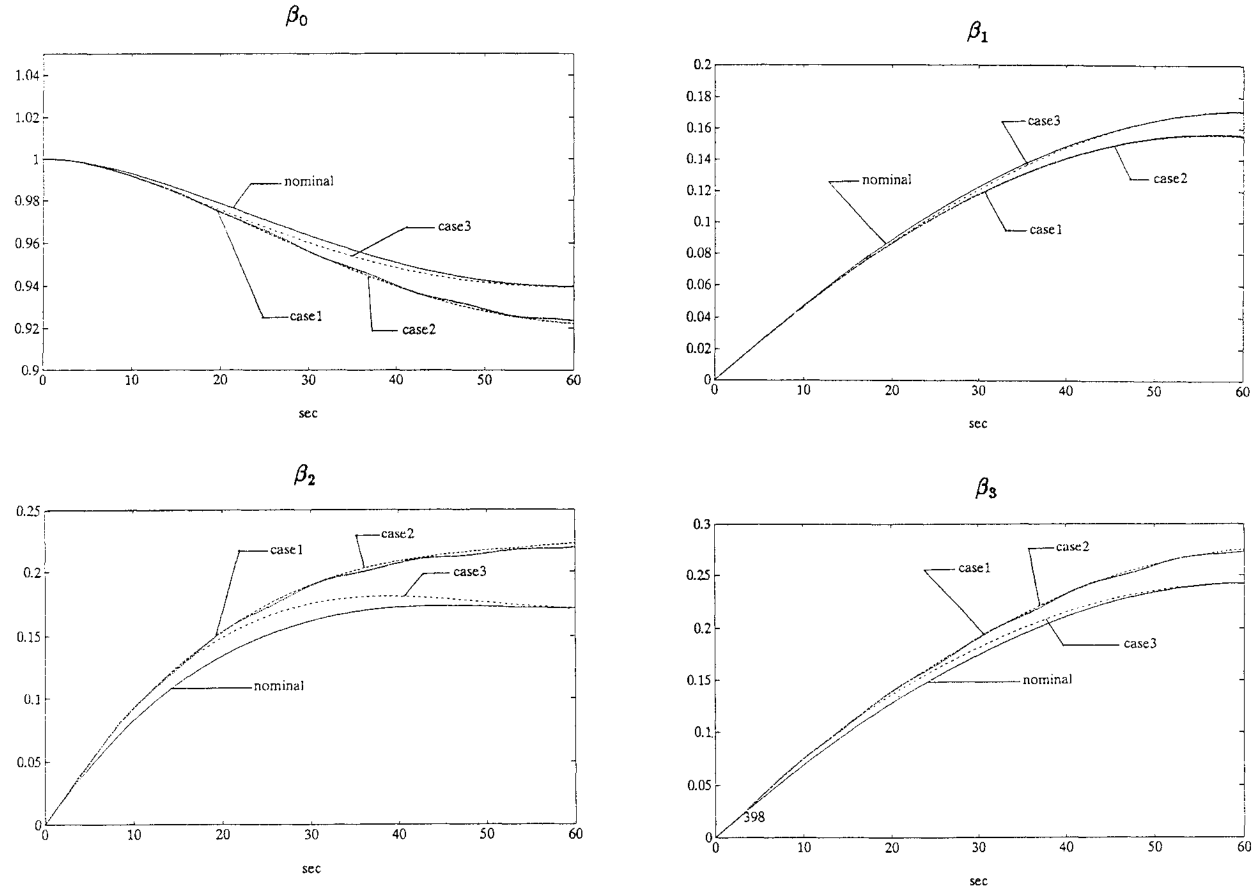

Fig. 3 Euler parameters of nominal, case 1, case 2, and case 3.

orienting the spacecraft as rigid in terms of minimum torque energy. The performance index J^0 and the system Hamiltonian H^0 are defined as

$$J^0 = \frac{1}{2}\int_0^{tf} \boldsymbol{T}^{\mathrm{T}} \underline{\underline{R}}_0 \boldsymbol{T}\,\mathrm{d}t, \qquad H^0 = \tfrac{1}{2}\boldsymbol{T}^{\mathrm{T}} \underline{\underline{R}}_0 \boldsymbol{T} + \boldsymbol{\lambda}^{\mathrm{T}}\dot{\boldsymbol{\zeta}}$$

where $\boldsymbol{T} = [T_x^0\ T_y^0\ T_z^0]^{\mathrm{T}}$ is the referencing optimal attitude torque vector and $\underline{\underline{R}}_0$ is the positive definite diagonal matrix; $\boldsymbol{\lambda}$ is the costate vector and $\zeta = [\omega_x^0\ \omega_y^0\ \omega_z^0\ \beta_0^0\ \beta_1^0\ \beta_2^0\ \beta_3^0]^{\mathrm{T}}$ representing rigid-body motions described by Eqs. (14–16) and (20). The computational procedure proposed in Ref. 11 is used here to find the optimal referencing trajectory. We considered using only the reaction wheels to generate the attitude torques.

Local Output Feedback Control

Local output feedback control is used in part 2 control to eliminate the structure vibrations. To this end, we need to discretize the distributed parameter system and reduce the system order for controller implementation. In the absence of body force term, Eqs. (22–24) are separable. Consider that the elastic quantities can be expressed as

$$u_{yi}^1(x,t) = \Theta_{yi}(x)q_{yi}(t),$$
$$u_{zi}^1(x,t) = \Theta_{zi}(x)q_{zi}(t), \qquad \phi_i^1(x,t) = \Theta_i(x)q_i(t) \tag{25}$$

where $\Theta_{yi}(x)$, $\Theta_{zi}(x)$, and $\Theta_i(x)$ are the shape functions, and $q_{yi}(x)$, $q_{zi}(x)$, and $q_i(x)$ are the generalized coordinates. Given an optimal nominal attitude solution and boundary conditions for the beams, we formulated the beam vibration equations by substituting Eq. (25) into Eqs. (22–24) and obtained

$$\ddot{\eta} + [C(t)]\dot{\eta} + [K(t)]\eta + [V(t)] = \boldsymbol{F}(t) \tag{26}$$

where $\eta = [q_{y1}\ q_{y2}\ q_{z1}\ q_{z2}\ q_1\ q_2]^{\mathrm{T}}$ for left and right beams; $[C(t)]$, $[K(t)]$, and $[V(t)]$ are known time-varying matrices with components associated to the linear operator and the reference trajectory; $\boldsymbol{F}(t)$ is the discretized local feedback control on the beams and is assumed in the form[3] of $\boldsymbol{F}(t) = [A]\dot{\eta} + [B]\eta + [V(t)]$, where $[A]$ and $[B]$ are the feedback gain matrices that can be derived through feedback control methods.

System Guidance

The guidance in part 3 is formulated as an optimal tracking problem. We define the guidance index J^1 to be minimized and the associated perturbed Hamiltonian H^1 as

$$J^1 = \frac{1}{2}\int_0^{t_f} \left(\delta\zeta^{\mathrm{T}} \underline{\underline{Q}} \delta\zeta + \delta\boldsymbol{T}^{\mathrm{T}} \underline{\underline{R}} \delta\boldsymbol{T}\right) \mathrm{d}t$$

$$H^1 = \tfrac{1}{2}\left(\delta\zeta^{\mathrm{T}} \underline{\underline{Q}} \delta\zeta + \delta\boldsymbol{T}^{\mathrm{T}} \underline{\underline{R}} \delta\boldsymbol{T}\right) + \delta\boldsymbol{\lambda}^{\mathrm{T}}\delta\dot{\zeta}$$

where $\delta\zeta = [\omega_x^1\ \omega_y^1\ \omega_z^1\ \beta_0^1\ \beta_1^1\ \beta_2^1\ \beta_3^1]$ and $\delta\boldsymbol{T} = [T_x^1\ T_y^1\ T_z^1]$ are the perturbed states and the perturbed torque vector of the hub, respectively; $\underline{\underline{Q}}$ and $\underline{\underline{R}}$ are symmetric semipositive and positive definite weighting matrices, respectively, and $\delta\boldsymbol{\lambda}$ is the costate. The optimal guidance has to satisfy the following first-order necessary conditions,[17]

$$\delta\dot{\zeta} = \frac{\partial H^1}{\partial \delta\boldsymbol{\lambda}}, \qquad \delta\dot{\boldsymbol{\lambda}} = -\underline{\underline{Q}}\delta\zeta - \frac{\partial}{\partial\delta\boldsymbol{\lambda}}(\delta\lambda^{\mathrm{T}}\delta\dot{\zeta})$$
$$\frac{\partial H^1}{\partial \delta\boldsymbol{T}} = \underline{\underline{R}}\delta\boldsymbol{T} + \frac{\partial}{\partial\delta\boldsymbol{T}}(\delta\lambda^{\mathrm{T}}\delta\dot{\zeta}) = 0 \tag{27}$$

where the first equation of (27) represents Eqs. (17–19) and (21). A given set of boundary conditions and the substitution of the third equation of (27) into the first two equations of (27) results in a linear time-varying two-point boundary-value problem. The problem can be solved with the method of particular solutions[11] in one single iteration as summarized in the Appendix. Once the trajectories of

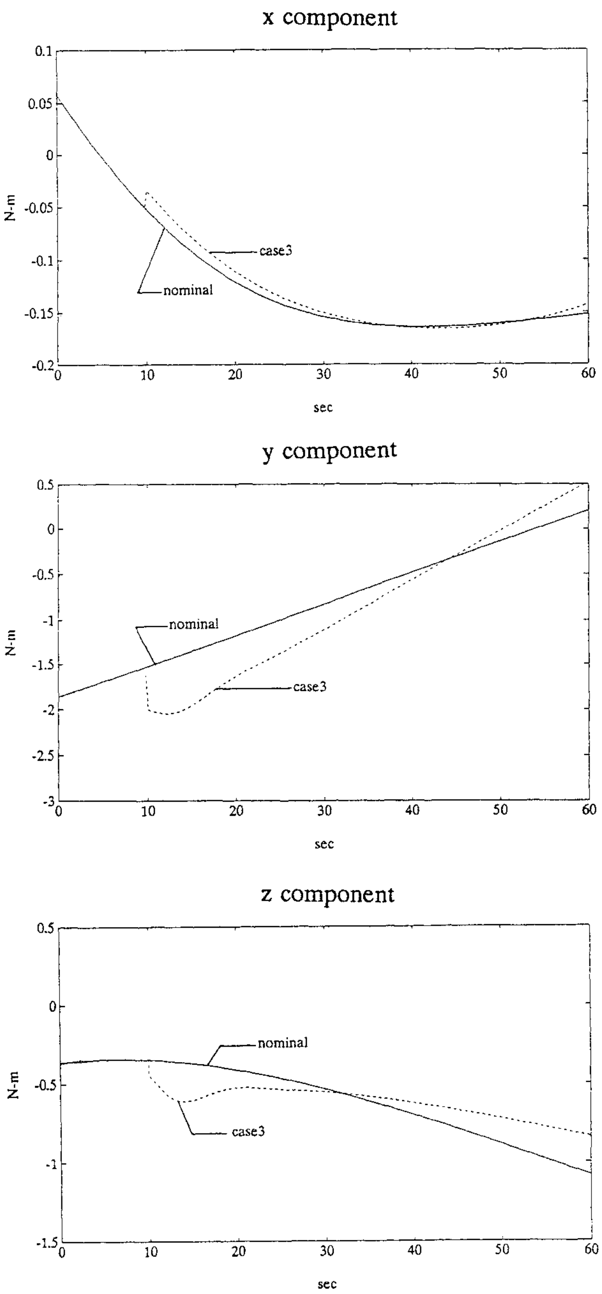

Fig. 4 Attitude control torques of the hub for nominal and case 3.

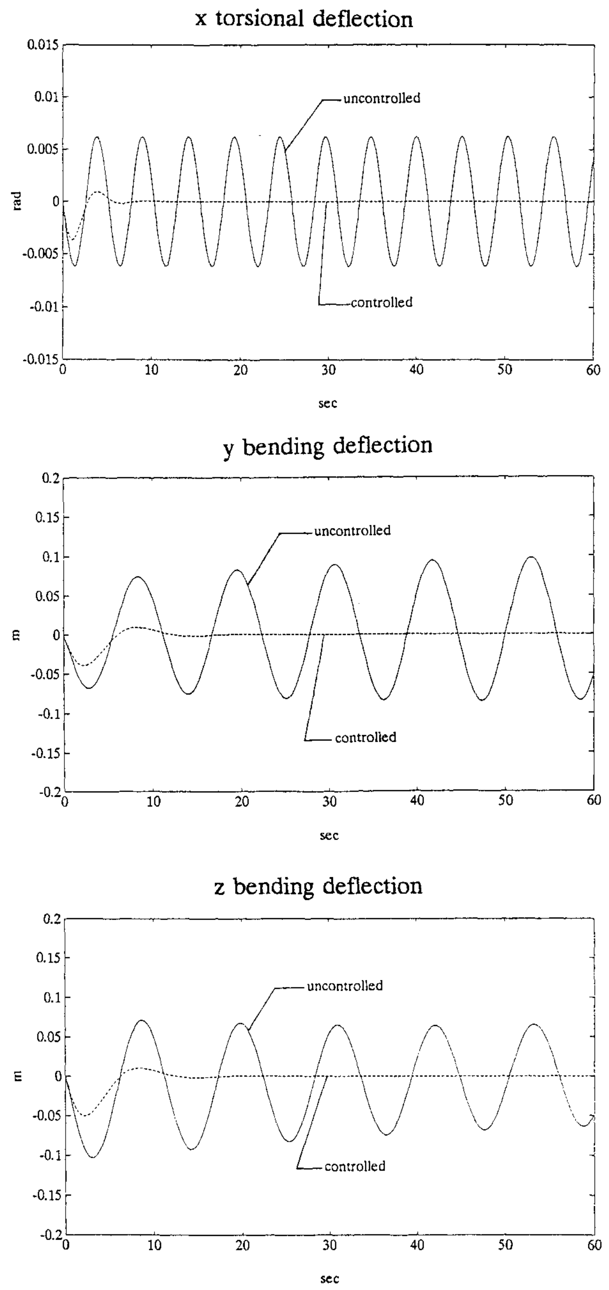

Fig. 5 Deflections of the tip of right-hand beam.

$\delta\zeta$ and $\delta\boldsymbol{\lambda}$ are obtained, the guidance torque $\delta\boldsymbol{T}$ can be computed from the third equation of (27). In reference to Eqs. (7–9), the guidance torques will be the resultant torques from the variations of the reaction wheels, environmental torques, and the torques induced by controlling the flexible beams. Although the guidance torques are found through numerical procedure, it can be regarded as feedback control function. For each time frame, the control is obtained in one single iteration, exactly like solving the differential Riccati equation. To formulate the Riccati equation, one makes the assumption that the system states are linearly proportional to the costates and a final penalty matrix is selected to integrate the time-varying differential Riccati equation backward. However, this assumption, which induces errors in the final boundary, is not quite accurate. Actually, the time-varying guidance torques are proportional to the costates,[17] which can be obtained by the optimality conditions represented by the third equation of (27).

Simulation

The following system parameters were assumed for the model spacecraft shown in Fig. 1:

System:

$$J_x = 1504\,\text{kg-m}^2, \qquad J_y = 2500\,\text{kg-m}^2, \qquad J_z = 2000\,\text{kg-m}^2$$

Beam:

$$L = 5\,\text{m}, \qquad \rho = 2\,\text{kg/m}, \qquad EI = 100\,\text{kg-m}^2$$

$$GJ = 20\,\text{kg-m}^2, \qquad J_p = 2\,\text{kg-m}^2$$

End mass:

$$m_e = 5\,\text{kg}, \qquad I_a = I_b = I_c = 2\,\text{kg-m}^2$$

The total mass of the hub is 1390 kg with radius of 1.2 m and height of 2.94 m. Attitude actuators and sensors were located on the hub, and the structure control actuators and sensors were placed at the tip of the beams. Three cases are included case 1 that applies part 1 open-loop reference control alone, case 2 that applies part 1 open-loop reference control and part 2 beams control, and case 3 that

Table 1 Final boundary conditions

State	Desired	Case 1	Case 2	Case 3
ω_x	0.00000	−0.00023	−0.00024	0.00000
ω_y	0.00000	0.00054	0.00051	0.00000
ω_z	0.00000	0.00036	0.00034	0.00000
β_0	0.93969	0.93494	0.93470	0.93926
β_1	0.17100	0.16628	0.16602	0.17154
β_2	0.17100	0.18503	0.18532	0.17172
β_3	0.24186	0.25326	0.25383	0.24240

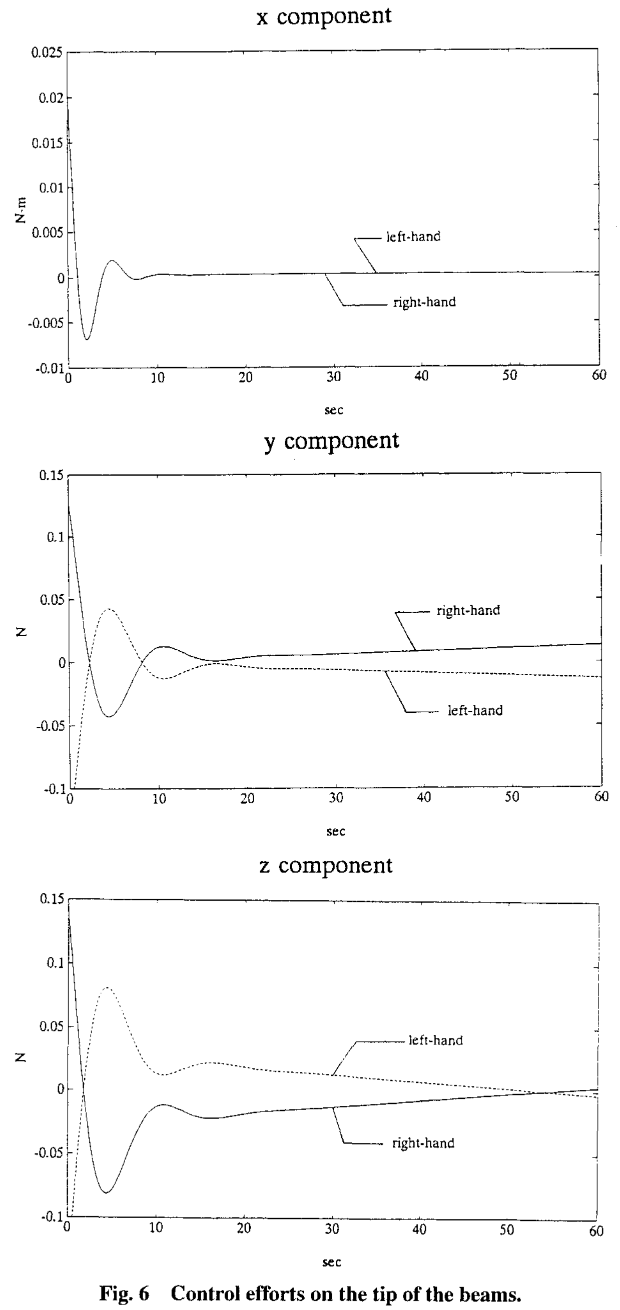

Fig. 6 Control efforts on the tip of the beams.

applies three-part composite control to the system. Because of the slow attitude motion, only the first mode of the beam was used as the other modes were assumed not excited or quickly damped. The spacecraft was to rotate about the body and inertial fixed principal rotational axis (0.5 0.5 0.7071) for 40 deg counterclockwise. The initial states of the spacecraft are assumed to be

$$\omega_x = 0.01, \qquad \omega_y = 0.02, \qquad \omega_z = 0.015 \quad \text{(rad/s)}$$

$$\beta_0 = 1, \qquad \beta_1 = 0, \qquad \beta_2 = 0, \qquad \beta_3 = 0$$

where the initial attitude rate may be produced by abrupt start of the reaction wheels combined with external disturbance torque excitations. A fixed 60-s maneuver time is specified for reorienting the spacecraft, which is about 1% of the orbital period at the altitude between 300 to 1000 km. Final states of the desired attitude, case 1, case 2, and case 3 are listed together in Table 1.

The simulated final boundaries of case 3, as shown in Table 1, indicate zero attitude rate and about 0.2% of attitude error were achieved. The error can be further reduced by reducing the present integration stepsize, which is 0.2 s.

Shown in Figs. 2 and 3 are the comparisons of attitude rates and attitude trajectories, respectively. Because of beam vibrations, the angular rate components in case 1 were oscillating along the nominal trajectory. The part 2 structure control was effective as shown in case 2 in eliminating the oscillatory motions but could not correct the attitude error. In case 3, the composite control was able to eliminate both the oscillatory motions and the attitude errors and followed the nominal trajectory to the specified final boundary. Attitude control torque components for nominal solution and for case 3 are shown in Fig. 4. Note that the control torque in case 3 began the guidance effort at 10 s to avoid transient motions of the beams. As the deflections are symmetric, only the tip motions of the right-hand side of the beam with and without local feedback control are compared in Fig. 5. The deflections are small because of small spacecraft angular rates. Bending and torsional frequencies of the uncontrolled beams under attitude maneuver were 0.56 and 1.21 rad/s, respectively. The structure control efforts are shown in Fig. 6. A feedback gain of $[A] = I$ and $[B] = \underline{0}$ were used in Eq. (25) for velocity feedback only. One may notice that even though the vibrations were quickly suppressed, the structure control efforts were not zero to the end of the maneuver. They were applied to remove the structure deflections induced from attitude motions. In the simulation, the coefficient matrices of performance indices used were $\underline{\underline{R}}_0 = I$, $\underline{\underline{Q}} = I$, and $\underline{\underline{R}} = I$, where I is the identity matrix and $\underline{0}$ is the zero matrix, with proper dimension.

Concluding Remarks

A three-part controller allowing simultaneous multi-axis large-angle maneuvers is proposed for spacecraft with flexible structures. Based on the configuration, each part of the controller can be designed and tuned independently. Although stability was not addressed, simulation results indicate that in case 1 the system under open-loop control oscillated about the reference trajectory and did not diverge within control period. Numerical results of case 3 also show that the derivation of one-way coupling from the rigid-body motions to the flexible-body motions and from the two motions to the attitude deviations is a good approximation for controller design. Under the effective control, rigid-body motions being assumed at least one order larger than the other two motions is indeed agreeable as shown in the figures. Unlike many studies that considered control torques on the hub only, the proposed method applies control torques to the hub and control forces and torques to the appendages. Complicated control arrangements are expected for eliminating excessive torques and forces, but the increase of number of the inputs implies the increase of controllability of the system. Instead of solving the Riccati equations, the numerical method is applied to find the time-varying guidance torque trajectory. From the computational standpoint, the proposed procedure is relatively stable and the results are quite accurate compared with numerically integrating the time-varying differential Ricatti equation with the large final penalty function.

Appendix: Numerical Solution of Optimal Guidance Trajectory

The numerical method of particular solutions[14] is efficient for solving a linear time-varying two-point boundary-value problem. To illustrate the method, we formulate the boundary-value problem first. Let the variables in Eq. (27) be defined as $x = [\delta\omega^{\mathrm{T}}\ \delta\beta^{\mathrm{T}}\ \lambda^{\mathrm{T}}]$, an $n \times 1$ vector, and Eq. (27) can be symbolically written as

$$\dot{x} = A(t)x \tag{A1}$$

where $A(t)$ is an $n \times n$ known time-varying matrix. We consider a more general problem by specifying the boundary conditions as

$$x_i(t_0) = a_i; \qquad i = 1, 2, \ldots, p \tag{A2}$$

$$x_j(t_f) = b_j; \qquad j = p+1, p+2, \ldots, n \tag{A3}$$

To solve the problem defined by Eqs. (A1–A3), one formulates $n-p+1$ base solutions of $\boldsymbol{x}^l$, $l = 1, 2, \ldots, n-p+1$, by integrating Eq. (A1) with the following initial conditions,

$$\begin{aligned} x_i^l(t_0) = a_i; \qquad & i = 1, 2, \ldots, p \\ l = 1, 2, \ldots, n-p+1 & \end{aligned} \tag{A4}$$

$$\begin{aligned} x_{p+k}^l(t_f) = \delta_{lk}; \qquad & k = 1, 2, \ldots, n-p \\ l = 1, 2, \ldots, n-p+1 & \end{aligned} \tag{A5}$$

where δ_{lk} is the Kronecker delta. Since Eq. (A1) is linear, the $n-p+1$ base solutions can be superimposed to obtain the solution of

$$\boldsymbol{x}(t) = \sum_{l=1}^{n-p+1} K_l \boldsymbol{x}^l(t) \tag{A6}$$

where K_l is the $n-p+1$ weighting coefficients that can be determined from the initial and the final boundary conditions. The boundary conditions give

$$\sum_{l=1}^{n-p+1} K_l = 1 \quad \text{and} \quad \underline{\underline{D}} \sum_{l=1}^{n-p+1} K_l \boldsymbol{x}^l(t_f) = \boldsymbol{b} \tag{A7}$$

where $\underline{\underline{D}}$ is an $n-p \times n$ coefficient matrix to equate the final boundary conditions of base solutions with $\boldsymbol{b}$, an $n-p \times 1$ vector representing Eq. (A3). The combined solution $\boldsymbol{x}(t)$ in Eq. (A6) satisfies both Eq. (A1) and the given boundaries.

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